Water Flow Estimation in One Open Channel with Data Assimilation

ME236
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April 24th, 2012
Outline

- Introduction
- Notations
- Special Methods with PDE
- Preliminary Results
- Conclusion
- Future work
Problem Description:
Real-time estimation of 1-D state in an open channel water flow networks with streaming data

Assumptions

- 1-D State: Discharge flow and Average depth
- Streaming Data: Noisy measurements at the boundaries and internal locations

Delta Simulation Model II (DSM2)
-one-dimensional mathematical model for dynamic simulation
Procedure

Process of data assimilation in open channel
Notations

Main Variables

\( H: \) Stage (ft)

\( V: \) Velocity (ft/s)

\( B: \) Channel Width (ft)

\( D: \) Hydraulic Diameter (ft)

\( S_0: \) Bed Slope (ft/ft)

\( S_f: \) Friction Slope (ft/ft)

Auxiliary Variables

\( Q: \) Flow Rate (ft\(^3\)/s)

\( A: \) Cross Sectional Area (ft\(^2\))

\( P: \) Parameter (ft)

\( m: \) Manning Coefficient (s/\( \sqrt[3]{ft} \))

\( \Delta t: \) Time Step Size (s)

\( \Delta x: \) Spatial Step Size (ft)
Governing Equations – Saint Venant Equations

\[ \frac{\partial H}{\partial t} + \frac{\partial VH}{\partial x} = 0 \]

\[ \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial H}{\partial x} = g(S_0 - S_f) \]

Boundary Conditions

\[ Q(0, t) = Q_u(t) \]
\[ H(L, t) = H_d(t) \]

Initial Conditions

\[ Q(x, 0) = Q_i(x) \]
\[ H(x, 0) = H_i(x) \]
The Sacramento Delta
Delta Simulation Model 2 (DSM2)
Discretization – Inner Nodes (Lax Diffusive Scheme)

\[
H_i^{k+1} = \frac{1}{2} (H_i^{k+1} + H_i^{k-1})
- \frac{\Delta t}{4\Delta x} (V_i^{i+1} + V_i^{i-1}) (H_i^{k+1} - H_i^{k-1})
- \frac{\Delta t}{4\Delta x} (D_i^{i+1} + D_i^{i-1}) (V_i^{i+1} - V_i^{i-1})
\]

\[
V_i^{k+1} = \frac{1}{2} (V_i^{i+1} + V_i^{i-1}) \left( 1 - \frac{\Delta t}{2\Delta x} (V_i^{k+1} - V_i^{k-1}) \right)
- \frac{g\Delta t}{2\Delta x} (H_i^{k+1} + H_i^{k-1})
+ g\Delta t \left( S_0 - \frac{S_{f,i+1}^{k} + S_{f,i-1}^{k}}{2} \right)
\]
Discretization – Boundary Nodes (Characteristics)

\[ H_{1}^{k+1} = H_{R}^{k} + C_{R}^{k} (V_{1}^{k+1} - V_{R}^{k}) + C_{R}^{k} \Delta t (S_{fR}^{k} - S_{0}) \]

\[ V_{N+1}^{k+1} = V_{L}^{k} + \frac{g}{C_{I}^{k}} (H_{L}^{k} - H_{N+1}^{k+1}) - g\Delta t (S_{fL}^{k} - S_{0}) \]

PA Sleigh, I M Goodwill, “The St Venant Equations”, School of Civil Engineering, University of Leeds, March 2000
Extended Kalman Filter

- Linearize the state equations about an estimation of the current mean and covariance
- Optimize the simulation results and improve the accuracy
- Time Update:

\[ \hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_{k-1}, 0) \]
\[ P_{k|k-1} = \phi_{k-1} P_{k-1|k-1} \phi_{k-1}^T + \varphi_{k-1} Q_{k-1|k-1} \varphi_{k-1}^T \]

- Measurement Update:

\[ K_k = P_{k|k-1} G_k^T (G_k P_{k|k-1} G_k^T + D_{k-1} R_k D_{k-1}^T)^{-1} \]
\[ \hat{y}_k = G_k \hat{x}_{k|k-1} \]
\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_k) \]
\[ P_{k|k} = (I - K_k G_k) P_{k|k-1} \]
Preliminary Results
We need to get this stuff fixed in time!!
Thank you!