Modeling Tree Sap Flow Using PDEs

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Introduction

- Determine whether using thermistors to measure the flow of tree sap is a potential alternative to current methods
- Model the thermistor as a point source of an instantaneous burst of heat
Notations
Problem Definition

• Determine the flow of tree sap using the Marshall Model and a known (measured) change in temperature

• The Marshall Model
  – Convection: \( \frac{dT}{dt} = u \frac{dT}{dx} \)
  – Conduction: \( \frac{dT}{dt} = \alpha \left( \frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} \right) \)
  – Heat Pulse: \( Q \delta(x)\delta(y)\delta(t) \)
Notations

• $u$ : tree sap velocity [cm/hr]
• $Q$ : heat source term [$^\circ\text{C} \cdot \text{cm}^2$]
• $W$ : change in temperature [$^\circ\text{C}$]
• $x$-$y$ plane : surface of trunk where $x$ is parallel to the trunk
• $t$ : time [s]
• $\alpha$ : thermal diffusivity of xylem [m$^2$/s]
Problem Solution Method

- Validate Marshall Model
  - Laplace Transform
  - Fourier Transform
  - Jump Condition
  - Solution
  - Inverse Laplace Transform
  - Branch Cut in the Complex Plane
  - Inverse Fourier Transform

\[ u = \frac{x}{t} - \frac{1}{t} \sqrt{-4\alpha t \ln \left( \frac{4W\pi\alpha t}{Q} \right)} - y^2 \]
Results and Simulations
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Future Steps

• Collect data from laboratory design and compare with Marshall Model
• Implement prototype in field tests