Arterial traffic

An integrated approach of distributed systems modeling and statistical models

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EE 291 (Spring 2010) – Aude Hofleitner
Recall: distributed systems modelling

Lighthill-Whitham-Richards partial differential equation
- Nonlinear first order hyperbolic scalar conservation law
- Concave flux function (empirical fundamental diagram)
- Weak boundary conditions
  \[\text{[Lighthill-Whitham, 1955; Richards, 1956]}\]

\[\rho(x, t) \text{ is the vehicle density.}\]

\[
\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0
\]

\[\rho(a, t) = \rho_a(t)\]
\[\rho(b, t) = \rho_b(t)\]
\[\rho(x, 0) = \rho_0(x)\]
In the case of arterial traffic

- A lot of **unknown** and highly **variable** parameters
  - Traffic lights
  - Pedestrians
  - Bad parking
  - Delivery trucks...
  - Capacity of the road
  - Different traffic flows (bikes, trucks...)
An integrated approach of distributed systems and statistical models

1. **Model the dynamics of the distributed system** (assumptions, LWR equation...)

2. Define a set of independent parameters $P$ describing the model (often have physical interpretations)

3. **Derive the probability distributions** of the state variables (density, velocity...) parameterized by $P$

4. When you observe data, estimate the parameters by maximizing the probability to observe the data: **maximize the likelihood**
   
   If you observe a lot of long travel times, long delays, the road is likely congested...
Step 1: Assumptions and model dynamics

- Neglect overtaking
- Stationarity of traffic conditions (constant arrival rate, fixed cycle timing, no constant increase or decrease of queue lengths)

- Triangular fundamental diagram
- Conservation of vehicles

\[
\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0
\]

\[
\rho(a, t) = \rho_a(t)
\]

\[
\rho(b, t) = \rho_b(t)
\]

\[
\rho(x, 0) = \rho_0(x)
\]
Step 2: Define the relevant parameters of the model

- Cycle timing (red and cycle time)
- Traffic conditions (arrival rate, queue length, clearing time)
- Driving behavior (free flow speed, distribution of free flow speed)

\[ \rho(x) \]

- Undersaturated regime
- Congested regime

\[ l_{\text{max}} + l_r \]

\[ \frac{1}{\rho_a} \]

\[ \frac{1}{\rho_c} \]
Step 3: Model the distribution of state variables

**Variable of interest:** density at location x, averaged over time

→ Probability distribution of observing a car at location x is proportional to the average density (normalizing constant)

→ Spatial heterogeneity of the travel time on arterials

→ Depends on the network and model parameters
Step 3: Model the distribution of state variable

**Variable of interest:** travel time. Depends on:
- the delay experienced,
- the *driving behavior*

→ Conditional probability distribution of travel time for a given free flow speed for a given set of parameters

→ Integrate over the free flow speeds (total probability law)
Step 3: Model the distribution of travel times

**Parametric travel time distributions, depends on**
- Origin a and destination b
- Parameters of the model

The distributions are quasi-concave (sublevel sets are convex).

*Proof not detailed here*
Step 4: estimate the parameters with data

**The likelihood function:**
- Probability of the observations conditioned on the value of the parameters,

**Maximize the log-likelihood:**
- Optimization problem
- Might have constraints dictated by the physics of the problem
  - e.g. bounds on the parameters, constraint on signals sharing an intersection, stationarity assumptions…

➔ Best estimate of the parameters, given the observed data
Step 4: estimate the parameters with data

• Solve the **constrained optimization** problem

\[
\begin{align*}
\text{maximize} & \quad \sum_i \ln(f(x_i)) \\
\text{s.t} & \quad \rho_a, \rho_{\text{max}}, \rho_c, \bar{v}_f, R, C, l_r, \alpha, \sigma^2 \\
& \quad \rho_a \leq \rho_c (1 - R/C) \\
& \quad l_r + \bar{v}_f (C - R) \rho_c / \rho_{\text{max}} \leq L
\end{align*}
\]

The constraints are given by the physics of the model (stationarity, cycle timing, queue length…)

- Estimation of the road parameters
- Spatial distribution of vehicles on a link: models the spatial heterogeneity of travel times
- Travel time distribution estimation and prediction
Results

- Needs little data
- Does not overfit
- Physical interpretation
- Estimation and prediction of travel times
- Improve accuracy (compare to baseline model)
Future work

- **For this project**: finalize the results for the travel time estimation and prediction

- **Ongoing work**:
  - Spatio temporal evolution of traffic conditions: transition probability matrices on the evolution of parameters (Hidden Markov Model) ➔ Summer 2010
  - Hierarchical model (bayesian analysis) ➔ Currently testing
  - Joint distribution of travel times on links with multiple intersections (flow synchronization)
Questions?

Thank you for your attention

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