Hydromorphology of an Urbanizing Watershed Using Multivariate Elasticity

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Acknowledgement: Richard Vogel, Tufts U.
Definitions and Details

• Hydromorphology: The structure and evolution of hydrologic systems.

• Question: How do hydrologic systems evolve in response to a variety of anthropogenic (human) and natural (climatic) influences?

• Goal: Better understand interactions between streamflow and trends in climate, land use, and water use.

• Case: Santa Ana River Watershed within Orange County
Santa Ana Watershed

2,500 mi.$^2$ total, 153.2 mi.$^2$ in Orange County
Data Sites
# Data Collection: 1940 - 1999

<table>
<thead>
<tr>
<th>Variable</th>
<th>Metric</th>
<th>Unit</th>
<th>Source</th>
<th>Resolution</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Streamflow</td>
<td>Discharge</td>
<td>cubic feet per second</td>
<td>USGS</td>
<td>daily average</td>
<td>21,900</td>
</tr>
<tr>
<td>Climate</td>
<td>Precipitation</td>
<td>1/100 inch</td>
<td>NOAA</td>
<td>monthly average</td>
<td>720</td>
</tr>
<tr>
<td>Water Use</td>
<td>Groundwater Levels</td>
<td>feet below ground surface</td>
<td>USGS</td>
<td>sporadic measurements</td>
<td>419</td>
</tr>
<tr>
<td>Land Use</td>
<td>Population</td>
<td>individuals</td>
<td>CDF</td>
<td>10-year census</td>
<td>7</td>
</tr>
</tbody>
</table>
Flow duration curve

- 1940s-1950s
- 1960s-1970s
- 1980s-1990s

Exceedance probability

Daily streamflow (cfs)

1940s-1950s 1960s-1970s 1980s-1990s
Natural Influences

Streamflow (cfs)

Precipitation (in/100)
Natural Influences

Streamflow (cfs)

Precipitation (in/100)

Precipitation Trend


Streamflow (cfs)

Precipitation (in/100)

Precipitation Trend

Natural Influences
Human Influences

![Graph showing groundwater depth and population trends over time.](image-url)
Elasticity

- Econometric concept applied to hydrology to describe sensitivity of streamflow to changes in other phenomena, for example precipitation:

The precipitation elasticity of streamflow, defined as

\[ \varepsilon_p = \frac{dQ/Q}{dP/P} = \frac{dQ}{dP} \frac{P}{Q} \]  \hspace{1cm} (1)

relates the proportional change in streamflow (at mean) to proportional change in precipitation. If \( \varepsilon_p = 2 \) for annual streamflows, then a 1% change in precipitation leads to a 2% change in streamflow.
Multivariate Elasticity

• Consider the total differential (Chapter 1) of streamflow resulting from simultaneous changes in precipitation (P), land use (L) and water use (W):

\[
dQ = \frac{\partial Q}{\partial P} dP + \frac{\partial Q}{\partial L} dL + \frac{\partial Q}{\partial W} dW
\]  

(2)

• It is useful to estimate the differentials as a percentage of change from the mean for each variable in (2):

\[
\begin{pmatrix}
\frac{Q-\bar{Q}}{\bar{Q}}
\end{pmatrix} = \frac{\partial Q}{\partial P} \bar{P} \begin{pmatrix}
\frac{P-\bar{P}}{\bar{P}}
\end{pmatrix} + \frac{\partial Q}{\partial L} \bar{L} \begin{pmatrix}
\frac{L-\bar{L}}{\bar{L}}
\end{pmatrix} + \frac{\partial Q}{\partial W} \bar{W} \begin{pmatrix}
\frac{W-\bar{W}}{\bar{W}}
\end{pmatrix}
\]

(3)

or

\[
q = \varepsilon_P \cdot p + \varepsilon_L \cdot l + \varepsilon_w \cdot w
\]  

(4)
Regression Analysis

• To find coefficients $\varepsilon_p, \varepsilon_L, \varepsilon_W$ use Ordinary Least Squares, which “fits” observed data, and returns standard errors and accuracy measurements. To do this, use datapoints $(x_i, y_i)$ with $i=1, 2, ..., n$ to find $f$ such that $y_i \approx f(x_i)$

...by minimizing $S = \sum_{i=1}^{n} (y_i - f(x_i))^2$

• Analysis performed on three streamflow conditions:
  – Average Streamflow $\bar{Q}$
  – Flood Condition $Q_{\text{max}}$
  – Low Flow Condition $Q_{90}$
## Results

<table>
<thead>
<tr>
<th>Streamflow Variable (in cfs)</th>
<th>Climate, $\varepsilon_p$</th>
<th>Land Use, $\varepsilon_L$</th>
<th>Water Use, $\varepsilon_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood - $Q_{max}$</td>
<td>$\varepsilon$ 1.749</td>
<td>0.664</td>
<td>-1.504</td>
</tr>
<tr>
<td></td>
<td>$S_{\varepsilon}$ 0.227</td>
<td>0.219</td>
<td>0.993</td>
</tr>
<tr>
<td></td>
<td>$p$ 0.000</td>
<td>0.004</td>
<td>0.135</td>
</tr>
<tr>
<td>Average - $\bar{Q}$</td>
<td>$\varepsilon$ 2.312</td>
<td>0.674</td>
<td>-2.116</td>
</tr>
<tr>
<td></td>
<td>$S_{\varepsilon}$ 0.413</td>
<td>0.397</td>
<td>1.803</td>
</tr>
<tr>
<td></td>
<td>$p$ 0.000</td>
<td>0.095</td>
<td>0.245</td>
</tr>
<tr>
<td>Drought - $Q_{90}$</td>
<td>$\varepsilon$ 2.857</td>
<td>0.694</td>
<td>-2.377</td>
</tr>
<tr>
<td></td>
<td>$S_{\varepsilon}$ 0.606</td>
<td>0.584</td>
<td>2.650</td>
</tr>
<tr>
<td></td>
<td>$p$ 0.000</td>
<td>0.240</td>
<td>0.373</td>
</tr>
</tbody>
</table>

The variables $\varepsilon$, $S_{\varepsilon}$, and $p$ are the elasticity estimate, its standard deviation, and p-value (confidence interval), respectively.
Goodness of Fit

Variation of Qbar, relative to mean (dimensionless)

Year


Modeled Streamflow

Measured Streamflow
Conclusions

About Data
- Real data can be problematic
- Climate has larger effect on average and low flows
- Land use has small effect but most on low flows
- Consistent with findings by others who have used this method

About Model
- Larger coefficients have greater accuracy (lower p-value)
- Any number of independent variables could be added to model
- Could be used to predict changes in development patterns or climate
Streamflow Prediction

• Population is expected to double over the next 50 years
  – Holding precipitation and water use constant…

  ➢ Daily Average Streamflow $\bar{Q}$ goes from 71 cfs to 110 cfs

• If precipitation increases with climate change (e.g. 20%)
  – Holding population and water use constant…

  ➢ Daily Average Streamflow $\bar{Q}$ goes from 71 cfs to 95 cfs