Convex function

• Convex functions
• Convex sets
• Reminder: derivative
• Gradient descent
• Problems of gradient descent

Convex function

$ f(x) = \lambda f(a) + (1- \lambda) f(b)$

$0 \leq \lambda \leq 1$

Non convex function

$ f(x) = \lambda f(a) + (1- \lambda) f(b)$

$0 \leq \lambda \leq 1$

Concave function

$ f(x) = \lambda f(a) + (1- \lambda) f(b)$

$0 \leq \lambda \leq 1$

Definitions / facts, common mistakes

Definitions/facts
• If f is convex, -f is concave
• If f is concave, -f is convex

Common mistakes
• A nonconvex function is not necessarily concave
• A nonconcave function is not necessarily convex
• A function can be neither concave nor convex
• A function can be concave and convex
Functions not defined for all x

Functions not defined for all x (convex)

Functions not defined for all x (convex)

Definition:
Convex set: for all $A$ and $B$ in the set, if $A$ and $B$ are in the set, $\lambda A + (1-\lambda)B$ is also in this set, for $0 \leq \lambda \leq 1$
Definition: Convex set: for all A and B in the set, if A and B are in the set, $A + (1-\lambda)B$ is also in this set, for $0 \leq \lambda \leq 1$. This set is not convex.

Convex sets

Famous convex sets

Famous non convex sets

Why do we use 'convex' for functions and sets

The epigraph (i.e. points above the graph) of a convex function is a convex set.

Convex sets and functions: basic properties

Local minimum might not be a global minimum

Local minimum is a global minimum
Convex sets and functions: basic properties

- Minimum might not be unique
- Convex function
- Local minimum is a global minimum

Convex optimization programs

\[
\text{min: } f(x) \\
\text{s.t. } g(x) \leq 0
\]

- \( f \) and \( g \) are convex functions, defined on convex sets
- Convex optimization programs are “easy” problems, compared to general optimization programs
- Local minimum is a global minimum

Reminder: derivative

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

The derivative of a function at a point represents its slope

Gradient descent (conceptual description)

You want to find the minimum of a function, starting from a guess, assuming that you cannot depict the graph of the function, for example the following function

\[
f(x) = \exp(\sin(x^2)) + \sqrt{x^4 + 3\sin(\exp(-\frac{1}{1+e^{|x|}}))}
\]

Idea:

1) Make a guess
2) Compute the derivative at this point (i.e. the slope)
3) Follow the direction of the slope (i.e. descend)
4) Stop when the slope is zero, i.e. it does not go downhill

Gradient descent (illustration)
Gradient descent (illustration)

Problem 1: non convex function
Problem 2: how to stop?

What if you cannot figure out where the stop point is?

Problem 3: how to hit the wall?

"algebraic minimum"

Problem 3: how to hit the wall?

"actual minimum"